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POSITION OF THE LAST POINT LOAD VARIABLE METHOD – WRITING BENDING MOMENT'S EXPRESSIONS

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ABSTRACT

This paper presents the use of the P.L.P.L. method to solve a system of a simply supported straight beam loaded by a convoy. It is the result of our own research that we have presented in thesis. Here, we make a highlight on the writing of bending moment's expressions. For vulgarization, we act to show the process and we convince to prove the ease offered by this method. We define in this paper, the aim of the use, which consists in determining the bending moments and the shear forces in the beam. We describe the model, then we present the general formula used to establish the expressions of bending moments under each load of the convoy. Thus, after describing all parameters related to the structure and those related to the loads, we show the expressions of bending moments in an abacus, which is shown as a table. This abacus would be used easily by those in charge of calculation. We show a case study of bridge where we present a program, on EXCEL, for writing bending moment's expressions. Then we show curves resulting from the expressions previously determined and we determine dangerous cross-section and also the dangerous position of the convoy on the beam. Thus after that we are able to calculate the maximum value of bending moment. We prove that the use of the method is simple, easy and leads to reliable results. In the future we will use the results of this research to solve a system of continuous beam loaded with a convoy.

KEY WORDS

P.LP.L Method, Simply supported beam; Convoy; Bending moment's expressions; Bridge

1- INTRODUCTION

The Position of the Last Point Load variable method (P.LP.L. variable Method) is the result of our research we have defended in thesis. We have published the results on MADAREVUES – Madamine [1]. This method consists of determining bending moments and share forces in a simply supported beam loaded by a convoy. So, all expressions of solicitations, depend mainly on the only variable « P.L.P.L. ». That's why we call this method as shown on the title. The determinations of bending moments and share forces due to a convoy on the beam are important for the choice of the dimensions of the structure. Because the strength of the structure is according to its resistance to these solicitations. We find this kind of situation in a case of a bridge or in a system of rolling charges.

Why we use this method on calculations? So this paper is written as a method's vulgarization which depends on the awareness of users about ease offered in the resolutions. The aim of the research is to provide facilities to students, technicians and engineers.

According to climate change, as mentioned in article in the International Journal of Energy, Environment, and Economics: « Energy efficiency is a national and international policy aimed at reducing energy consumption and CO_2 emissions to achieve energy security » [2]. So we think, our research contributes in saving energy, on reducing times of studies process by making facilities in the determinations. In another way, our study concerns calculation of bridges, and we think also that bridge contributes in resilience against climate change by the way that it is used to pass through up floods.

We write this article as follows: in a first time, we describe the method and its principle; in a second time we present the results; and in a third time we develop a case study that shows the ease of use of the method, particularly about writing bending moment's expressions. We conclude the main parts of this paper with the discussions.

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2- METHOD

2-1. Definition

This method concerns a standard modeling. So we relied on the basic principles of the Mechanics of Structures using the Cross-Section method. We consider also the different parameters of the structure, those related to the convoy as well as the Position of the Last Point Load (P.L.P.L.) variable witch is the only variable of determination. We note this variable as « x_1 ». All expressions of stresses are written in relation of the unique variable and in relation of the defined parameters related to the simply supported beam and related to the characteristics of the convoy. The position « x_1 » is important in the method because it make the resolve easy according to be the only variable of determination.

2-2. Model describing

The model uses mainly parameters related to the simply supported beam, those related to the characteristics of the convoy and the P.L.P.L. variable itself for the determination of the applied bending moments and also share forces. We describe below the parameters used in this method:

L : Simply supported beam length.

l_i : Distances between each load of the convoy.

P_i : Values of the convoy loads.

R : Resultant load « R ».

x'_R : Position of the Resultant load in reference to the first load of the convoy.

x_1 : Position of the Last Point Load Variable.

Note that, in the numerical modeling for the use of the method, it may be that the number of loads of the convoy could be considered. We note this parameters as « n ». The Figure 1 show the model.

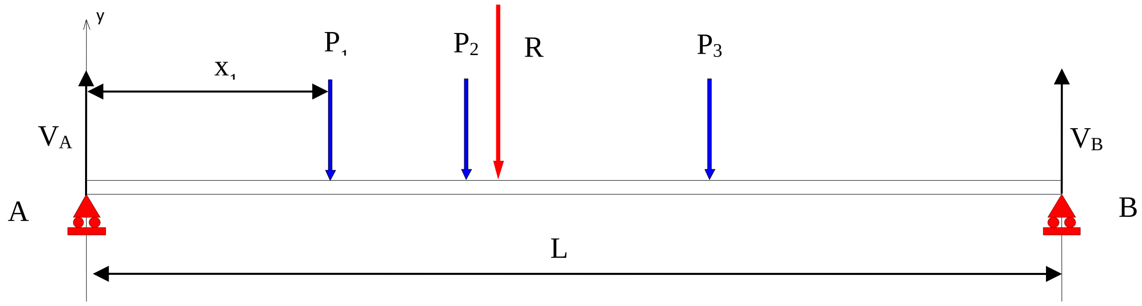

Figure 1: Model showing

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To show a real example defining these parameters, we take the model of a bridge charged by two 30tons truck of B_c type with six axles according to the Fascicule 61[3]. This example is developed in the case study, in the Discussion's part.

3- RESULTS

3-1. General formula

Simple and generalized expression could be written. So, in our research, we present the general formula.

$$M_{f/i} = A(x_1) + B_i(x_1) + C_i \quad (5\text{-}1)$$

In the general formula, the parts « A » and « B » are based on « x_1 ». The part « C » concerns independent constants not related to « x_1 ».

3-2. Bending moment's expressions

The general formula allows us to define expressions under each load of the convoy. The Table 1, presents the expressions of bending moment under the « P_i » loads. We show in this table six expressions corresponding to six charges. But the general formula could be used to write more expressions if there are more charge in the convoy.

The part « A » is the one that recurs in bending moment expressions corresponding to each load of the convoy.

$$\left(R.x_1 - \frac{R}{L}.x_1.x_1 - \frac{R}{L}.x_1.x'_R\right)$$

And the part « B » includes the sub-expressions, incorporating the distances between each load of the convoy. It changes, but accumulates, for each load of the convoy. So for « P_2 », we have:

$$\left(R.l_1 - \frac{R}{L}.l_1.x_1 - \frac{R}{L}.l_1.x'_R\right)$$

The Part C is built with constant part, as for « P_2 », we have:

$$(-P_1 . l_1)$$

Expressions in the next table are function of second degree of « x_1 », with the different parameters related to the characteristic of the beam and to the characteristics of the convoy.

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Table 1: Bending moment's expressions under P_i

Under P_1	Under P_2	Under P_3
$\left(R.x_1-\frac{R}{L}.x_1.x_1-\frac{R}{L}.x_1.x'_R\right)$	$\left(R.x_1-\frac{R}{L}.x_1.x_1-\frac{R}{L}.x_1.x'_R\right)$ + $\left(R.l_1-\frac{R}{L}.l_1.x_1-\frac{R}{L}.l_1.x'_R\right)$ + $(-P_1\ .\ l_1)$	$\left(R.x_1-\frac{R}{L}.x_1.x_1-\frac{R}{L}.x_1.x'_R\right)$ + $\left(R.l_1-\frac{R}{L}.l_1.x_1-\frac{R}{L}.l_1.x'_R\right)$ + $\left(R.l_2-\frac{R}{L}.l_2.x_1-\frac{R}{L}.l_2.x'_R\right)$ + $(-P_1\ .\ l_1)+(-P_1\ .\ l_2)+(-P_2\ .\ l_2)$
Under P_4	Under P_5	Under P_6
$\left(R.x_1-\frac{R}{L}.x_1.x_1-\frac{R}{L}.x_1.x'_R\right)$ + $\left(R.l_1-\frac{R}{L}.l_1.x_1-\frac{R}{L}.l_1.x'_R\right)$ + $\left(R.l_2-\frac{R}{L}.l_2.x_1-\frac{R}{L}.l_2.x'_R\right)$ + $\left(R.l_3-\frac{R}{L}.l_3.x_1-\frac{R}{L}.l_3.x'_R\right)$ + $(-P_1\ .\ l_1)+(-P_1\ .\ l_2)+(-P_1\ .\ l_3)$ + $(-P_2\ .\ l_2)+(-P_2\ .\ l_3)$ + $(-P_3\ .\ l_3)$	$\left(R.x_1-\frac{R}{L}.x_1.x_1-\frac{R}{L}.x_1.x'_R\right)$ + $\left(R.l_1-\frac{R}{L}.l_1.x_1-\frac{R}{L}.l_1.x'_R\right)$ + $\left(R.l_2-\frac{R}{L}.l_2.x_1-\frac{R}{L}.l_2.x'_R\right)$ + $\left(R.l_3-\frac{R}{L}.l_3.x_1-\frac{R}{L}.l_3.x'_R\right)$ + $\left(R.l_4-\frac{R}{L}.l_4.x_1-\frac{R}{L}.l_4.x'_R\right)$ + $(-P_1\ .\ l_1)+(-P_1\ .\ l_2)+(-P_1\ .\ l_3)$ + $(-P_1\ .\ l_4)+(-P_2\ .\ l_2)+(-P_2\ .\ l_3)$ + $(-P_2\ .\ l_4)+(-P_3\ .\ l_3)+(-P_3\ .\ l_4)$ + $(-P_4\ .\ l_4)$	$\left(R.x_1-\frac{R}{L}.x_1.x_1-\frac{R}{L}.x_1.x'_R\right)$ + $\left(R.l_1-\frac{R}{L}.l_1.x_1-\frac{R}{L}.l_1.x'_R\right)$ + $\left(R.l_2-\frac{R}{L}.l_2.x_1-\frac{R}{L}.l_2.x'_R\right)$ + $\left(R.l_3-\frac{R}{L}.l_3.x_1-\frac{R}{L}.l_3.x'_R\right)$ + $\left(R.l_4-\frac{R}{L}.l_4.x_1-\frac{R}{L}.l_4.x'_R\right)$ + $\left(R.l_5-\frac{R}{L}.l_5.x_1-\frac{R}{L}.l_5.x'_R\right)$ + $(-P_1\ .\ l_1)+(-P_1\ .\ l_2)+(-P_1\ .\ l_3)$ + $(-P_1\ .\ l_4)+(-P_1\ .\ l_5)+(-P_2\ .\ l_2)$ + $(-P_2\ .\ l_3)+(-P_2\ .\ l_4)+(-P_2\ .\ l_5)$ + $(-P_3\ .\ l_3)+(-P_3\ .\ l_4)+(-P_3\ .\ l_5)$

		$+$ $(-P_4 \,.\, l_4) + (-P_4 \,.\, l_5) + (-P_5 \,.\, l_5)$

4- DISCUSSIONS

To continue with discussions, we will show a case study of a bridge loaded with a convoy. Because this case illustrates well the use of the P.LP.L. Method.

4-1. Case study

We study the case of the determination of bending moments due to a convoy, on the main beams of a two-lane bridge. Beams are straight and simply supported with 20 m length.

The bridge is loaded on each lane by a convoy of two 30 tons trucks of B_c type, according to the Fascicule 61 [3]. The convoy consists of six axle loads « P_i » with their respective spacing « l_i ». We consider that the beams form a single set of bars that will support the whole of the system of loads as shown on Figure 2.

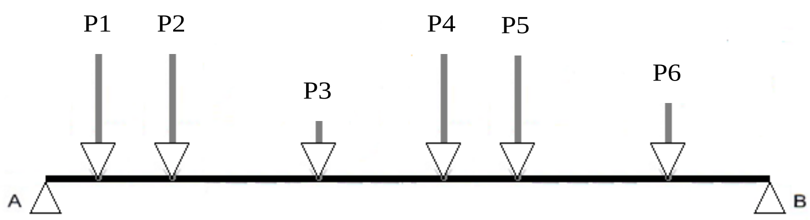

Figure 2: Beam charged by a convoy of 6 point loads

4-1-1. Writing bending moment's expressions

A simple tool modeled on EXCEL is presented for the transformation of parametric equations. We present data, specifying the parameters related to the convoy and related to the simply supported beam. Table 2 shows parameters needed for the calculations. They concern the values of « P_i », the spacing « l_i », the resultant load « R » with its position « x'_R » and finally the span « L » of the simply supported beam.

Table 2: Presentation of parameters on EXCEL

R	120	t	l_1	1,5	m	P_1	24	t
L	20	m	l_2	4,5	m	P_2	24	t
x'_R	7,05	m	l_3	4,5	m	P_3	12	t
			l_4	1,5	m	P_4	24	t
			l_5	4,5	m	P_5	24	t
						P_6	12	t

Thus, after data inputs, we use the general formula previously presented to clearly distinguish the three parts of each expression, mainly the parts A, B and C. We enter into each calculation cell the formulas corresponding to the expressions, identifying the coefficients in the sub-expressions in front of « x_1^2 », the coefficients in the sub-expressions in front of « x_1 » and also the independent constants.

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Table 3, shows calculation related to the coefficients in the sub-expressions and the independent constants.

Table 3: EXCEL model of determination of coefficients and constants

A	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1
	0	180 -9 x_1 -63,45	180 -9 x_1 -63,45	180 -9 x_1 -63,45	180 -9 x_1 -63,45	180 -9 x_1 -63,45
		-36	540 -27 x_1 -190,35	540 -27 x_1 -190,35	540 -27 x_1 -190,35	540 -27 x_1 -190,35
B			-252	540 -27 x_1 -190,35	540 -27 x_1 -190,35	540 -27 x_1 -190,35
				-522	180 -9 x_1 -63,45	180 -9 x_1 -63,45
					-648	540 -27 x_1 -190,35
C						-1134

We finish with displaying the final bending moment's expressions under each load of the convoy in the Table 4.

Table 4: Displaying bending moment's expressions on EXCEL

Sous P_1	**Sous P_2**	**Sous P_3**	**Sous P_4**	**Sous P_5**	**Sous P_6**
-6 x_1^2	-6 x_1^2	-6 x_1^2	-6 x_1^2	-6 x_1^2	-6 x_1^2
77,7 x_1	68,7 x_1	41,7 x_1	14,7 x_1	5,7 x_1	-21,3 x_1
	80,55	214,2	293,85	284,4	148,05

4-1-2. Resolution – Maximum values of bending moments

For all loads, the bending moment's expressions are summarized below:

$$M_1 = -6.x_1{}^2 + 77{,}7.x_1$$

$$M_2 = -6.x_1{}^2 + 68{,}7.x_1 + 80{,}55$$

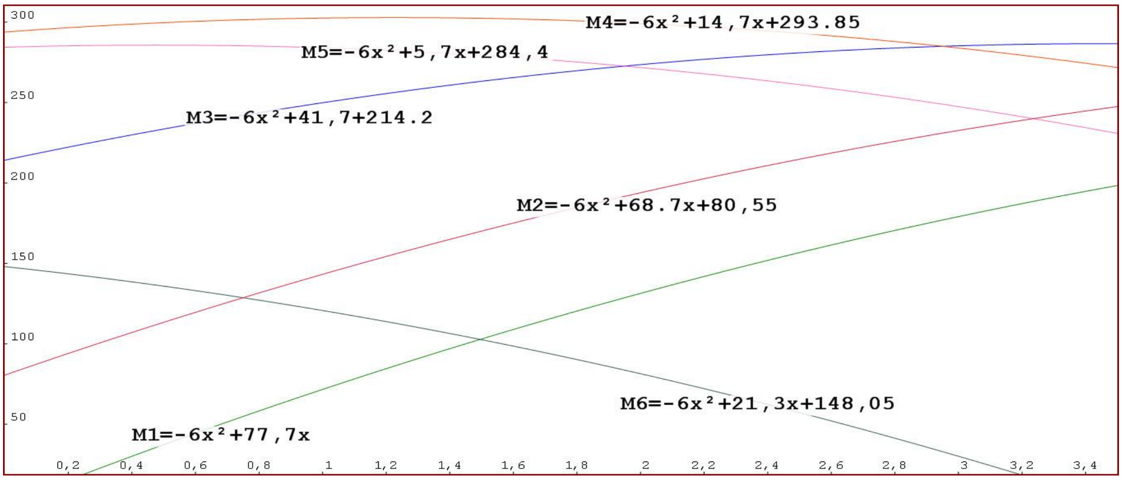
$$M_3 = -6.x_1{}^2 + 41{,}7.x_1 + 214{,}2$$

$$M_4 = -6.x_1{}^2 + 14{,}7.x_1 + 293{,}85$$

$$M_5 = -6.x_1{}^2 + 5{,}7.x_1 + 284{,}4$$

$M_6 = -6.x_1^2 + 21{,}3.x_1 + 148{,}05$

Then with these expressions, we get the envelope curves, which can be drawn for each load of the convoy, as we show on Figure 3.



In abscissa : Values of « x_1 » in m *In ordinate : Values of bending moments in ton-m*

Figure 3: Bending moment's envelop curves

And after determination, we conclude that the cross-section under the load « P_4 » presents the maximum value of bending moment with a value of 302,854 ton-m when « x_1 =1,225 m ». The share forces are determined by deriving the expressions of moments according to the P.L.P.L. variable.

4-1-3. Comparison with other method

In previous studies, the results were compared with those obtained in using other methods. So, after showing the determination with the use of the influence line and the calculation with Autodesk Structural Analysis, we have conclude, in our publication [1], that we get the same results.

4-2. Findings and analysis

The curves, in previous figure, show that for each position according to « x_1 », we can determine each value of bending moment under each load of the convoy, in its motion along the beam; which is not the case, using some other methods. As we said before, the results of using this method are the same issued to the using of others. Thus we can say that the determination by the P.L.P.L. variable is simple, particularly the way for writing bending moment's expressions. And this method is reliable and lead to a good precision in calculations. Today, the method is not well known because it is a new one. That’s why we act for its vulgarization toward people susceptible to use it.

5- CONCLUSION

The use of the P.L.P.L. variable method offers a simple way of determination. By identifying the parameters of the model, we can find the bending moment's expressions. Then we can determine the unfavorable cross-section, also the unfavorable position of the convoy on the beam and finish with the calculations of bending moments. The P.L.P.L. method improve the resolution on making simplicity, precision and reliability. The use of this method offers rapidity in the process of the structure concept and analysis related to the study of a beam loaded with a convoy. We could solve general cases, including the case of a convoy with several loads of different values with varied spacing's. In the next time, we will use the results of this research to solve a system of continuous beam loaded with a convoy.

6- ACKNOWLEDGEMENTS

We thank Professor Christian RAKOTONIRINA, for the discussions and fruitful exchanges that contributed to the writing of this paper.

7- REFERENCES

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[3] Fascicule 61, *Conception, calcul et épreuves des ouvrages d'art*.

8- TABLES

Table 1: Bending moment's expressions under P_i

Under P_1	Under P_2	Under P_3
$\left(R.x_1 - \frac{R}{L}.x_1.x_1 - \frac{R}{L}.x_1.x'_R\right)$	$\left(R.x_1 - \frac{R}{L}.x_1.x_1 - \frac{R}{L}.x_1.x'_R\right)$ $+$ $\left(R.l_1 - \frac{R}{L}.l_1.x_1 - \frac{R}{L}.l_1.x'_R\right)$ $+$ $(-P_1 . l_1)$	$\left(R.x_1 - \frac{R}{L}.x_1.x_1 - \frac{R}{L}.x_1.x'_R\right)$ $+$ $\left(R.l_1 - \frac{R}{L}.l_1.x_1 - \frac{R}{L}.l_1.x'_R\right)$ $+$ $\left(R.l_2 - \frac{R}{L}.l_2.x_1 - \frac{R}{L}.l_2.x'_R\right)$ $+$ $(-P_1 . l_1) + (-P_1 . l_2) + (-P_2 . l_2)$
Under P_4	**Under P_5**	**Under P_6**
$\left(R.x_1 - \frac{R}{L}.x_1.x_1 - \frac{R}{L}.x_1.x'_R\right)$ $+$ $\left(R.l_1 - \frac{R}{L}.l_1.x_1 - \frac{R}{L}.l_1.x'_R\right)$ $+$ $\left(R.l_2 - \frac{R}{L}.l_2.x_1 - \frac{R}{L}.l_2.x'_R\right)$ $+$ $\left(R.l_3 - \frac{R}{L}.l_3.x_1 - \frac{R}{L}.l_3.x'_R\right)$ $+$ $(-P_1 . l_1) + (-P_1 . l_2) + (-P_1 . l_3)$ $+$ $(-P_2 . l_2) + (-P_2 . l_3)$ $+$ $(-P_3 . l_3)$	$\left(R.x_1 - \frac{R}{L}.x_1.x_1 - \frac{R}{L}.x_1.x'_R\right)$ $+$ $\left(R.l_1 - \frac{R}{L}.l_1.x_1 - \frac{R}{L}.l_1.x'_R\right)$ $+$ $\left(R.l_2 - \frac{R}{L}.l_2.x_1 - \frac{R}{L}.l_2.x'_R\right)$ $+$ $\left(R.l_3 - \frac{R}{L}.l_3.x_1 - \frac{R}{L}.l_3.x'_R\right)$ $+$ $\left(R.l_4 - \frac{R}{L}.l_4.x_1 - \frac{R}{L}.l_4.x'_R\right)$ $+$ $(-P_1 . l_1) + (-P_1 . l_2) + (-P_1 . l_3)$ $+$ $(-P_1 . l_4) + (-P_2 . l_2) + (-P_2 . l_3)$ $+$ $(-P_2 . l_4) + (-P_3 . l_3) + (-P_3 . l_4)$ $+$ $(-P_4 . l_4)$	$\left(R.x_1 - \frac{R}{L}.x_1.x_1 - \frac{R}{L}.x_1.x'_R\right)$ $+$ $\left(R.l_1 - \frac{R}{L}.l_1.x_1 - \frac{R}{L}.l_1.x'_R\right)$ $+$ $\left(R.l_2 - \frac{R}{L}.l_2.x_1 - \frac{R}{L}.l_2.x'_R\right)$ $+$ $\left(R.l_3 - \frac{R}{L}.l_3.x_1 - \frac{R}{L}.l_3.x'_R\right)$ $+$ $\left(R.l_4 - \frac{R}{L}.l_4.x_1 - \frac{R}{L}.l_4.x'_R\right)$ $+$ $\left(R.l_5 - \frac{R}{L}.l_5.x_1 - \frac{R}{L}.l_5.x'_R\right)$ $+$ $(-P_1 . l_1) + (-P_1 . l_2) + (-P_1 . l_3)$ $+$ $(-P_1 . l_4) + (-P_1 . l_5) + (-P_2 . l_2)$ $+$ $(-P_2 . l_3) + (-P_2 . l_4) + (-P_2 . l_5)$ $+$

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		$(-P_3 \,.\, l_3) + (-P_3 \,.\, l_4) + (-P_3 \,.\, l_5)$ $+$ $(-P_4 \,.\, l_4) + (-P_4 \,.\, l_5) + (-P_5 \,.\, l_5)$

Table 2: Presentation of parameters on EXCEL

R	120	t	l_1	1,5	m	P_1	24	t
L	20	m	l_2	4,5	m	P_2	24	t
x'_R	7,05	m	l_3	4,5	m	P_3	12	t
			l_4	1,5	m	P_4	24	t
			l_5	4,5	m	P_5	24	t
						P_6	12	t

Table 3: EXCEL model of determination of coefficients and constants

A	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1	120 x_1 -6 x_1^2 -42,3 x_1
B	0	180 -9 x_1 -63,45	180 -9 x_1 -63,45	180 -9 x_1 -63,45	180 -9 x_1 -63,45	180 -9 x_1 -63,45
		-36	540 -27 x_1 -190,35	540 -27 x_1 -190,35	540 -27 x_1 -190,35	540 -27 x_1 -190,35
			-252	540 -27 x_1 -190,35	540 -27 x_1 -190,35	540 -27 x_1 -190,35
				-522	180 -9 x_1 -63,45	180 -9 x_1 -63,45
					-648	540 -27 x_1 -190,35
C						-1134

Table 4: Displaying bending moment's expressions on EXCEL

Sous P_1	Sous P_2	Sous P_3	Sous P_4	Sous P_5	Sous P_6
-6 x_1^2	-6 x_1^2	-6 x_1^2	-6 x_1^2	-6 x_1^2	-6 x_1^2
77,7 x_1	68,7 x_1	41,7 x_1	14,7 x_1	5,7 x_1	-21,3 x_1
	80,55	214,2	293,85	284,4	148,05

9- FIGURES

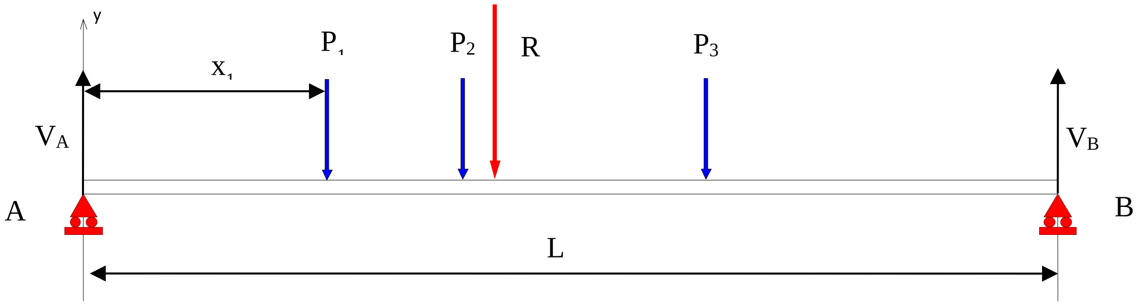


Figure 4: Model showing

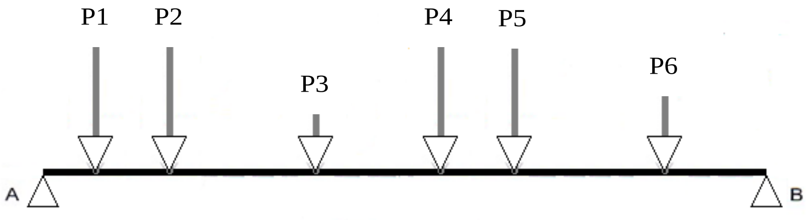
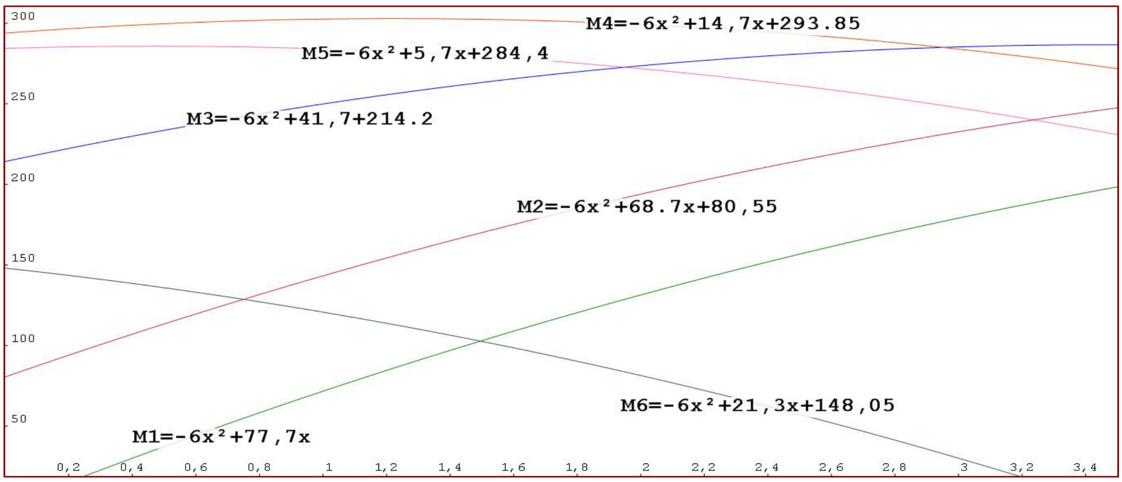


Figure 5: Beam charged by a convoy of 6 point loads



In abscissa : Values of « x_1 » in m *In ordinate : Values of bending moments in ton-m*

Figure 6: Bending moment's envelop curves